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Interaction due to a mechanical source in transversely isotropic micropolar media

Ibrahim A Abbas[1,2] and Rajneesh Kumar[3]

Abstract

The present investigation is concerned with interaction due to a mechanical source in transversely isotropic micropolar elastic media, determined using the finite element method. A particular type of normal force has been taken to illustrate the utility of the approach. The components of displacement, stress and microrotation are obtained and depicted graphically for a specific model. A special case of interest is also deduced from the present investigation.

Keywords

Transversely isotropic, micropolar media, finite element method

1. Introduction

Classical mechanics deals with the basic assumption that the effect of the microstructure of a material is not essential for describing mechanical behavior. Such an approximation has been shown in many well-known cases. Often, however, discrepancies between the classical theory and experiments are observed, indicating that the microstructure might be important. For example, discrepancies have been found in the stress concentrations in the areas of holes, notches and cracks; elastic vibrations characterized by high-frequency and small wavelengths, particularly in granular composites consisting of stiff inclusions embedded in a weaker matrix, fibers or grains; and the mechanical behavior of complex fluids, such as liquid crystals, polymeric suspensions and animal blood. In general, granular composites, for example porous materials, are widely used in the area of passive noise control as sound absorbers, and the effect of acoustical waves characterized by high frequencies and small wavelengths become significant.

To explain the fundamental departure of microcontinuum theories from the classical continuum theories, a continuum model embedded with microstructures to describe the microscopic motion or a nonlocal model to describe the long range material interaction is developed. This theory extends the application of the continuum model to microscopic space and short time scales. Micromorphic theory (Suhubi and Eringen, 1964; Eringen, 1999) treats a material body as a continuous collection of a large number of deformable particles, with each particle possessing finite size and inner structure. Using assumptions such as infinitesimal deformation and slow motion, micromorphic theory can be reduced to Mindlin's microstructure theory (Mindlin, 1964). When the microstructure of the material is considered rigid, it becomes the micropolar theory (Eringen, 1966).

Eringen's micropolar theory is more appropriate for geological materials such as rocks and soils, since this theory takes into account the intrinsic rotation and predicts the behavior of material with an inner structure. Different researchers have discussed different types of problems in transversely isotropic elastic material. Abubakar (1962) discussed free vibrations of a transversely isotropic plate. Suvalov et al. (2005) described the long-wavelength onset of the fundamental branches for a free anisotropic plate with arbitrary through-plate variation of material properties. Payton (1992) has

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studied wave propagation in a restricted transversely isotropic elastic solid whose slowness surface contains conical points. Tomar (2005) and Kumar and Deswal (2006) studied some problems of wave propagation in micropolar elastic media with voids. However, no attempt has been made to study the deformation in micropolar transversely isotropic material using the finite element method.

The exact solution of the governing equations of the micropolar generalized thermoelastic theory for a coupled and nonlinear/linear system exists only for very special and simple initial and boundary problems. A numerical solution technique is used to calculate the solution of general problems. For this reason the finite element method is chosen.

The finite element method is a powerful technique originally developed for the numerical solution of complex problems in structural mechanics, and it remains the method of choice for complex systems. A further benefit of this method is that it allows physical effects to be visualized and quantified regardless of experimental limitations. Abbas and Abd-alla (2008), Abbas and Palani (2010), Abbas and Othman (2011, 2012), Othman and Abbas (2011, 2012) and Abbas (2012a, 2012b) have successfully applied the finite element method to various problems in generalized thermoelastic materials.

The aim of the present study is to enhance our knowledge about the application of the finite element method in a micropolar transversely isotopic media. This study has many applications in various fields of science and technology, namely, atomic physics, industrial engineering, thermal power plants, submarine structures, pressure vessels, aerospace, chemical pipes and metallurgy.

The present investigation determines the components of displacements, microrotation and stress due to a mechanical source in transversely isotropic micropolar media. Such mechanical normal loading may produce a severe deformation in a thin zone near the half-space surface and thereby cause excessive wear and even cracking near the contact zone. It is therefore useful to analyze this class of problems by using a formulation that is as exact as possible and to provide results for the surface and/or near-surface field quantities (displacements, microrotation, stresses) that may be required for design purposes. The solution to the problem investigated here has practical applications in the fields of geomechanics, engineering, fiber-wound composites and laminated composite materials.

2. Basic equations

Following Eringen (1999), the constitutive relations and balance laws in general micropolar anisotropic medium possessing a center of symmetry, in the absence of body forces and body couples, are given by

Constitutive relations:

$$\sigma_{ij} = A_{ijkl}E_{kl} + G_{ijkl}\Psi_{kl},$$
$$m_{ij} = G_{ijkl}E_{kl} + B_{ijkl}\Psi_{kl}, \tag{1}$$

The deformation and wryness tensor are defined by

$$E_{ji} = u_{i,j} + \varepsilon_{ijkl}\phi_k, \quad \Psi_{ij} = \phi_{i,j}.$$

Balance laws:

$$\sigma_{ji,j} = \rho\ddot{u}_i,$$
$$m_{ik,i} + \varepsilon_{ijk}t_{ij} = \rho j\ddot{\phi}_k, \tag{2}$$

where σ_{ji} and m_{ij} are, respectively, the stress tensor and couple stress tensor, ρ is bulk mass density and u_i and ϕ_i are, respectively, the components of the displacement vector and microrotation vector. $A_{ijkl}, G_{ijkl}, B_{ijkl}$ are characteristic constants of material following the symmetry properties given by Eringen (1999).

3. Formulation of the problem

We have used appropriate transformations, following Slaughter (2002), on the set of Equations (1) to derive equations for the micropolar transversely isotropic medium. In the present paper, we consider a homogeneous, transversely isotropic micropolar elastic half-space $x \geq 0$. A continuous normal force is assumed to be acting on the micropolar elastic half-space, as shown in Figure 1. All the considered functions will depend on the time t and the coordinates x and z. Thus we assume that the displacement vector and microrotation vector are, respectively, of the form

$$\vec{u} = (u(x,z),\ 0,\ w(x,z)),\ \vec{\phi} = \left(0,\ \phi_y(x,z),\ 0\right). \tag{3}$$

Making use of (3) in Equations (1) and (2) yields

$$A_{11}\frac{\partial^2 u}{\partial x^2} + A_{55}\frac{\partial^2 u}{\partial z^2} + (A_{13}+A_{56})\frac{\partial^2 w}{\partial x\partial z} + K_1\frac{\partial\phi_y}{\partial z} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{4}$$

$$A_{66}\frac{\partial^2 w}{\partial x^2} + A_{33}\frac{\partial^2 w}{\partial z^2} + (A_{13}+A_{56})\frac{\partial^2 u}{\partial x\partial z} + K_2\frac{\partial\phi_y}{\partial x} = \rho\frac{\partial^2 w}{\partial t^2}, \tag{5}$$

$$B_{77}\frac{\partial^2\phi_y}{\partial x^2} + B_{66}\frac{\partial^2\phi_y}{\partial z^2} - X\phi_y - K_1\frac{\partial u}{\partial z} - K_2\frac{\partial w}{\partial z} = \rho j\frac{\partial^2\phi_y}{\partial t^2}, \tag{6}$$

where

$$K_1 = A_{56} - A_{55}, \quad K_2 = A_{66} - A_{56}, \quad X = K_2 - K_1.$$

For further considerations, it is convenient to introduce the dimensionless variables defined by

$$\begin{aligned}(x', z') &= \frac{\omega^*}{c_1}(x, z), (u', w') = \frac{\omega^*}{c_1}(u, w), t'_{ij} = \frac{t_{ij}}{A_{11}},\\ m'_{ij} &= \frac{m_{ij}c_1}{B_{66}\omega^*}\phi'_y = \frac{\phi_y A_{55}}{K_1}, t' = \omega^* t, \omega^{*2} = \frac{X}{\rho j},\\ c_1^2 &= \frac{A_{11}}{\rho}.\end{aligned} \tag{7}$$

Using Equation (7) in Equations (2)–(6), we obtain the equations in nondimensional form after suppressing the primes as

$$\frac{\partial^2 u}{\partial x^2} + d_1\frac{\partial^2 u}{\partial z^2} + d_2\frac{\partial^2 w}{\partial x \partial z} + d_3\frac{\partial \phi_y}{\partial z} = \frac{\partial^2 u}{\partial t^2}, \tag{8}$$

$$d_4\frac{\partial^2 w}{\partial x^2} + d_5\frac{\partial^2 w}{\partial z^2} + d_2\frac{\partial^2 u}{\partial x \partial z} + d_6\frac{\partial \phi_y}{\partial x} = \frac{\partial^2 w}{\partial t^2}, \tag{9}$$

$$d_7\frac{\partial^2 \phi_y}{\partial x^2} + d_8\frac{\partial^2 \phi_y}{\partial z^2} - d_9\phi_y - d_{10}\frac{\partial u}{\partial z} - d_{11}\frac{\partial w}{\partial x} = \frac{\partial^2 \phi_y}{\partial t^2}, \tag{10}$$

$$\sigma_{zz} = d_{12}\frac{\partial u}{\partial x} + d_5\frac{\partial w}{\partial z}, \tag{11}$$

$$\sigma_{xx} = \frac{\partial u}{\partial x} + d_{12}\frac{\partial w}{\partial z}, \tag{12}$$

$$\sigma_{xz} = d_{13}\frac{\partial w}{\partial x} + d_4\frac{\partial u}{\partial z} + d_{14}\phi_y, \tag{13}$$

$$m_{xy} = d_{15}\frac{\partial \varphi_y}{\partial x}, \tag{14}$$

where

$$(d_1, d_2, d_3, d_4, d_5, d_6) = \frac{1}{A_{11}}\left(A_{55}, A_{13} + A_{56}, \frac{K_1^2}{A_{55}}, A_{66}, A_{33}, \frac{K_1 K_2}{A_{55}}\right),$$

$$\begin{aligned}(d_7, d_8) &= \frac{1}{\rho j c_1^2}(B_{77}, B_{66}), \quad (d_9, d_{10}) = \frac{1}{\rho j \omega^{*2}}(X, A_{55}), d_{11} = \frac{K_2}{K_1}d_{10},\\ (d_{12}, d_{13}, d_{14}) &= \frac{1}{A_{11}}(A_{13}, A_{56}, K_1), d_{15} = \frac{B_{77}k_1}{B_{66}A_{55}}\end{aligned}$$

4. Initial and boundary conditions

The above equations, solved subject to initial conditions and boundary conditions, are

$$u = w = \phi_y = 0, \dot{u} = \dot{w} = \dot{\phi}_y, t = 0, x \geq 0, -\infty < z < \infty, \tag{15}$$

$$\begin{aligned}&t_{xx} = -f_0 H(t)(2l - |z|), \quad t_{xz} = 0, \quad m_{xy} = 0,\\ &x = 0, \; t > 0, \; -\infty < z < \infty,\end{aligned} \tag{16}$$

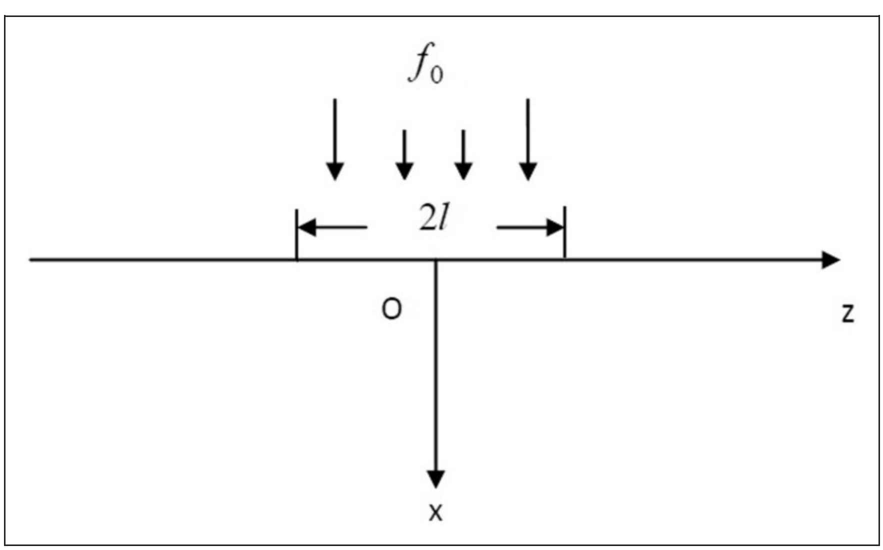

Figure 1. Geometry of the problem.